Modeling Drifter Motion

Anthony C. Kriezis

Project Supervisors: Thomas R. Consi, Dixia Fan
Faculty Supervisor: Michael S. Triantafyllou

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Contents

Abstract

In this UROP I studied the hydrodynamic properties of drifters. First of all, I printed small scale models of a drifter, using different wing configurations (one solid, one with two large holes, one with 32 small holes) and used the towing tank to measure the drag and from it calculate the coefficient of drag. The third model was found to have the highest coefficient. Afterwards, the coefficient of added mass was determined by oscillating the drifter in the towing tank. Once this was done, the force on the drifter was analyzed using a drag component and an added mass component. The resulting differential equation was solved using mathematica and the results were plotted on a coordinate system and a map, leading to a model that can predict the motion of the drifter in the water.

1 Introduction

One of the biggest projects of Sea Grant involves studying the impact of climate change on the acidity of the oceans and its effects on the creatures that live there. For this reason it is developing a fleet of drifters that will be deployed in the sea and collect data on the acidity of the water. There are two main types of drifters, the Coastal Ocean Dynamics Experiment (CODE) drifter, which uses large wings to follow the currents, and the Surface Velocity Program (SVP) drifter, which instead of wings uses a large drogue to follow the currents. Since the nature of the research involves measuring the pH primarily in coastal waters and lakes, which are shallow bodies of water, the SVP drifter was inappropriate and thus a CODE drifter was used.
The purpose of this UROP was to study the hydrodynamics of the motion of a CODE drifter. I began by designing small scale models of drifters with different wing configurations and running them in the towing tank. The results were subsequently analyzed to find the coefficient of drag and therefore determine which shape is the most favorable. The second stage of the project required an analysis of the equations governing the motion of the drifter and the determination of all the coefficients. Once this was done, a model was developed that predicts the motion of the drifter in the sea given some environmental data.

2 Testing Different Shapes

2.1 Designing and printing

The first stage of the UROP involved the creation of different models of the drifter that would be used to study the effect of different shapes on the coefficient of drag. The purpose was to determine which shape has the highest coefficient of drag, which will result in a high drag force and will allow the drifter to more closely follow the currents. All models were designed in a scale of $10 : 1$ with the actual Sea Grant drifter. Thus the height of each model was 15cm, and each of the four wings were $10cm$ by $5cm$. In addition, a square plate was added in the middle to preserve structural rigidity, noting that it would not affect the overall motion since it was perpendicular to it. Finally, the top cylinder was not drawn to scale (it had a diameter of $0.5in$) so as to connect it to the towing tank.
Three models were created in total, and the difference between them was the shape of the wings. The first model (Figure 1), which corresponds to the original drifter design, had rigid wings with no holes on them. The surface area when moving with two wings facing would thus be $100cm^2$.
The second model (Figure 2) was designed so that each of the four wings would have two large holes. The diameter of the holes was such that the surface area of the wings would be 90% of the original, i.e. $90cm^2$.
The third model (Figure 3) was designed so that each of the four wings would

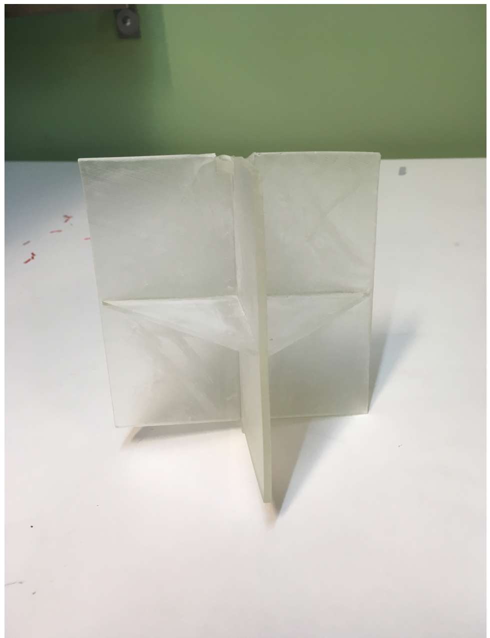


Figure 1: Drifter 1 (Dim. $10cm \times 10cm$)

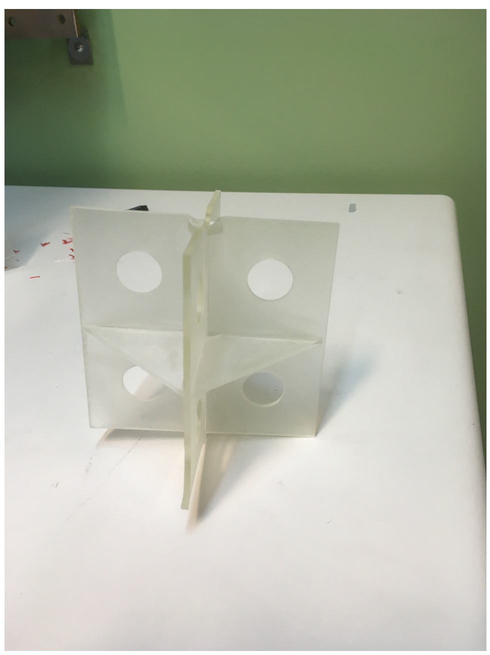


Figure 2: Drifter 2 (Dim. $10cm \times 10cm$)

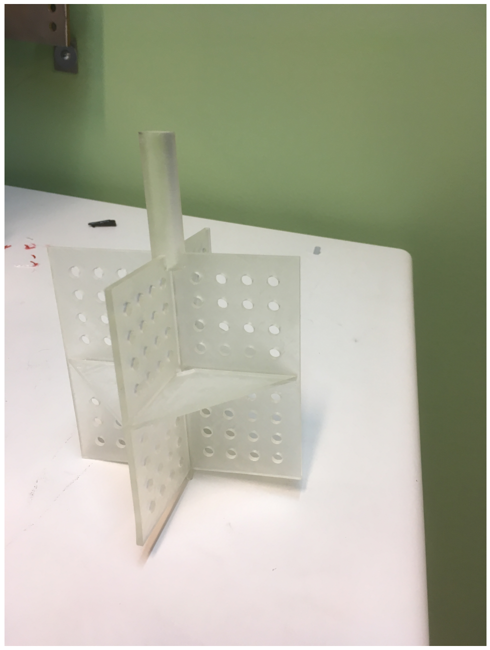


Figure 3: Drifter 3 (Dim. $10cm \times 15cm$)

have 32 small holes on them. Again, the surface area was designed to be 90% of the original, i.e. $90cm^2$.

All three models were printed using the Form 2 stereolithography 3D printer. The material was clear V4 resin.

2.2 Calculating Drag

The three models were then towed in the towing tank. All three models were towed 8 times, two with a speed of $0.10\frac{m}{s}$, two with $0.15\frac{m}{s}$, two with $0.20\frac{m}{s}$, and two with $0.25\frac{m}{s}$. In addition, for each model this set was repeated with the drifter making a 45° angle with the line of motion. The speeds were chosen to be three times smaller than the speeds the actual drifter will encounter, so that the model and the actual drifter have the same Froude number.

To find the coefficient of drag, the drag equation was used

$$F = \frac{1}{2} \cdot C_d \cdot \rho \cdot S \cdot v^2 \tag{1}$$

where F is the drag force, C_d is the coefficient of drag, ρ is the density, S is the surface area, and v is the velocity.

This equation can be solved for the drag coefficient. The results are given in the three tables (Figures 4-6). The first column shows the number of the trial, the second the mean force that was measured, the third the mean velocity of

Trial	Mean Force Fx	Mean Velocity	Actual Velocity	Drag Coefficient		Predicted Force
1	0.0823	0.1	0.3	1.646		67.17898626
2	0.078	0.1	0.3	1.56		67.17898626
3	0.1668	0.15	0.45	1.482666667		151.1527191
4	0.1641	0.1501	0.4503	1.456723721		151.3543232
5	0.2899	0.2	0.6	1.4495		268.7159451
6	0.2916	0.2	0.6	1.458		268.7159451
7	0.4503	0.25	0.75	1.44096		419.8686641
8	0.4532	0.2501	0.7503	1.449080504		420.2046263
			Mean	1.492866361		

Figure 4: Results for drifter 1

Trial	Mean Force Fx	Mean Velocity	Actual Velocity	Drag Coefficient		Predicted Force
1	0.0724	0.1	0.3	1.608888889		58.59505923
2	0.0671	0.1	0.3	1.491111111		58.59505923
3	0.1406	0.1498	0.4494	1.39235244		131.4875473
4	0.1474	0.15	0.45	1.455802469		131.8388833
5	0.2508	0.2	0.6	1.393333333		234.3802369
6	0.251	0.2	0.6	1.394444444		234.3802369
7	0.3928	0.25	0.75	1.396622222		366.2191202
8	0.4055	0.25	0.75	1.441777778		366.2191202
			Mean:	1.446791586		

Figure 5: Results for drifter 2

the model drifter, the fourth the actual velocity it would correspond to, the sixth the coefficient that is calculated, and the eighth the predicted force that the drifter would encounter if it moved with the actual velocity.

We can see that although the second drifter's coefficient is less than the original's (Figures 4, 5), the coefficient for the third drifter (Figure 6) is larger than the original's by a considerable amount (almost 10%). This result shows that the design of a drifter with many small holes, like a grater, experiences higher drag and is therefore superior to the original design of a continuous sheet according to which drifters are currently built.

3 Developing a model

3.1 Finding the equations

Having determined the ideal shape and measured the coefficient of drag, the question arose whether we could predict the course of the drifter in the water. To analyze that Newton's second law $F = m \cdot a$ was used. In the case of the drifter the main forces that govern its motion are the drag force $F = \frac{1}{2} \cdot C_d \cdot \rho \cdot S \cdot v^2$ and the added mass force $F = C_m \cdot \rho \cdot V \cdot a$, where C_m is the coefficient of added mass, ρ is the density, V is the submerged volume, and

Trial	Mean Force Fx	Mean Velocity	Actual Velocity	Drag Coefficient		Predicted Force
1	0.0819	0.1	0.3	1.82		65.66409646
2	0.0809	0.1	0.3	1.797777778		65.66409646
3	0.1591	0.1501	0.4503	1.569264974		147.941275
4	0.1654	0.15	0.45	1.633580247		147.744217
5	0.2758	0.2	0.6	1.532222222		262.6563858
6	0.2793	0.2	0.6	1.551666667		262.6563858
7	0.4255	0.2496	0.7488	1.517741777		409.0883716
8	0.4348	0.2498	0.7494	1.548432056		409.7442246
			Mean:	1.621335715		

Figure 6: Results for drifter 3

a is the acceleration. Another point that should be considered before the equations are used is that the added mass force results from the difference between the acceleration of an object with the neighboring fluid, and similarly the drag force arises from a difference between the object's and the fluid's velocity. Therefore the equations should be modified to become
$F = C_m \cdot \rho \cdot V \cdot (r'' - a)$ for the added mass force and
$F = \frac{1}{2} \cdot C_d \cdot \rho \cdot S \cdot |r' - v| \cdot (r' - v)$ for the drag force, where a and v denote the acceleration and velocity of the water and I have written the drag force in vector form.

$$m \cdot r'' = -C_m \cdot \rho \cdot V \cdot (r'' - a) - \frac{1}{2} \cdot C_d \cdot \rho \cdot S \cdot |r' - v| \cdot (r' - v) \tag{2}$$

where r is the position vector and m is the mass of the drifter.
This equation can be decomposed into two differential equations in the x and y directions. The solution to these equations can be used to trace the path of the drifter.
In this equation, m, V, ρ are known, C_d has been determined, and a, v are variable properties of the water and should be determined by measurements. Therefore the only constant in need of measurement is the coefficient of added mass C_m.

3.2 Measuring the coefficient of added mass

To measure the coefficient of added mass, the drifter was oscillated in the towing tank according to

$$x = A \cdot sin(\omega \cdot t) \tag{3}$$

Then we have

$$x' = A \cdot \omega \cdot sin(\omega \cdot t) \tag{4}$$

$$x'' = -A \cdot \omega^2 \cdot sin(\omega \cdot t) \tag{5}$$

Our equation of motion is

$$F = -C_m \cdot \rho \cdot V \cdot x'' - \frac{1}{2} C_d \cdot \rho \cdot S \cdot x' \cdot |x'| \tag{6}$$

If we integrate this over one period we notice that the drag term of the equation vanishes (since $\int_0^T sin^2(\omega \cdot t) \cdot cos(\omega \cdot t)dt = 0$). Multiplying by x”), we have

$$\int_0^T F \cdot x'' dt = \int_0^T -C_m \cdot \rho \cdot V \cdot (A^2 \cdot \omega^4 \cdot sin^2(\omega \cdot t)dt \tag{7}$$

$$\int_0^T F \cdot x'' dt = -C_m \cdot \rho \cdot V \cdot A^2 \cdot \omega^4 \cdot \int_0^T sin^2(\omega \cdot t)dt \tag{8}$$

$$\int_0^T F \cdot x'' dt = -C_m \cdot \rho \cdot V \cdot A^2 \cdot \omega^4 \cdot \frac{T}{2} dt \tag{9}$$

From here we can solve for C_m:

$$C_m = -\frac{\int_0^T F \cdot x'' dt}{\rho \cdot V \cdot A^2 \cdot \omega^4 \cdot \frac{T}{2}} \tag{10}$$

We can approximate the integral on the RHS by:

$$\int_0^T F \cdot x'' dt = \sum_{i=1}^{n} F_i \cdot x_i'' \cdot \Delta t \tag{11}$$

Thus

$$C_m = -\frac{\sum_{i=1}^{n} F_i \cdot x_i'' \cdot \Delta t}{\rho \cdot V \cdot A^2 \cdot \omega^4 \cdot \frac{T}{2}} \tag{12}$$

and if we assume $T = n \cdot \Delta t$

$$C_m = -\frac{2}{n} \cdot \frac{\sum_{i=1}^{n} F_i \cdot x_i''}{\rho \cdot V \cdot A^2 \cdot \omega^4} \tag{13}$$

A more accurate calculation of the added mass is obtained if we replace $\rho \cdot V$ in the denominator with $\frac{1}{2} \cdot \frac{4}{3} \cdot \pi \cdot R^3 \cdot \rho$, where R is the radius of a circle that has the same area as the drifter. In our case we can approximate R=5cm. Our equation therefore becomes:

$$C_m = -\frac{2}{n} \cdot \frac{\sum_{i=1}^{n} F_i \cdot x_i''}{\frac{1}{2} \cdot \frac{4}{3} \cdot \pi \cdot R^3 \cdot \rho \cdot A^2 \cdot \omega^4} \tag{14}$$

In order to compute the coefficient of added mass, the drifter was towed, and the measured force times the acceleration over one period were added, and so the coefficient of added mass was determined. The drifter was oscillated six times, two with amplitude $2cm$, two with $5cm$, and two with $8cm$, and with a constant frequency of $0.75Hz$. The results are shown below (Figure 7). Shown below are also graphs of the measured force and acceleration when the amplitude is $2cm$ (Figures 8,9).

Trial	Amplitude/h	Cm
1a	8	0,166802
1b	8	0,191016
2a	20	0,465741
2b	20	0,492979
3a	32	0,18947
3b	32	0,110056
	Mean	0,269344

Figure 7: Coefficient of added mass

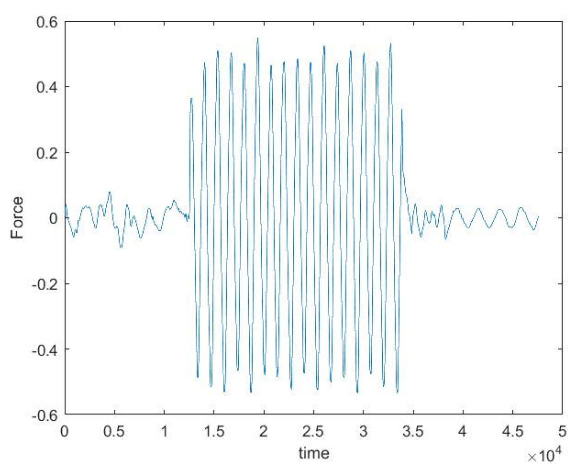


Figure 8: Measured force when $A = 2cm$

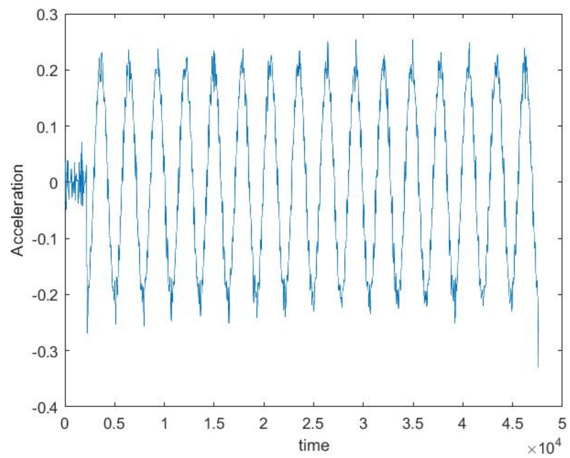


Figure 9: Measured acceleration when $A = 2cm$

3.3 Creating the model

Having calculated the coefficient of added mass, all the pieces are in place to create a model that will predict the motion of the drifter. The model was developed using the Mathematica envoronment.
The code, shown below, includes an initialization of parameters (I have used the mass and volume of the model drifter), an initialization of environmental parameters (to be collected from sensors), and the initial conditions of the drifter. It then uses the function NDSolve to find a numerical solution to this system of ODEs, which is then plotted using ParametricPlot. The trajectory that is produced by the following conditions is shown below (Figure 10), along with a plot of the speed in the y direction (Figure 11).

```
ms = 0.06599  (*kg*);
Cm = 1.068558;
rho = 1000 (*kg/m^3*);
Cd = 8.605549496;
S = 0.01 (*m^2*);
V = 0.00006599 (*m^3*);
(*Specify conditions of water*)
ay = 0;
ax = 0;
vy = 0.1;
vx = 0.5;
(*Specify initial conditions*)
init1 = y[0] == 0;
init2 = x[0] == 0;
init3 = y'[0] == 0;
```

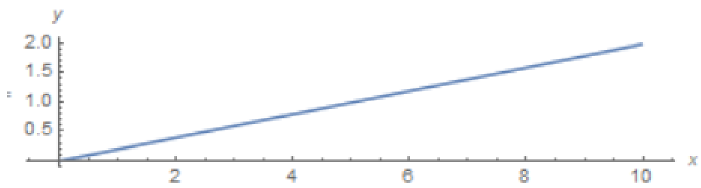


Figure 10: Trajectory of Drifter

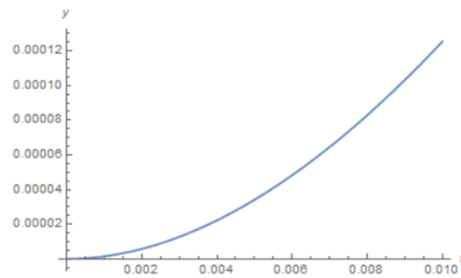


Figure 11: y versus t

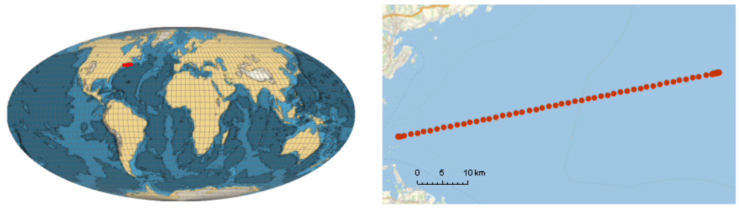
```
init4 = x'[0] == 0;

ode1 = ms*y''[t] == -Cm*rho*V*(y''[t] - ay) -
    0.5*Cd*rho*S*Abs[y'[t] - vy]*(y'[t] - vy);
ode2 = ms*x''[t] == -Cm*rho*V*(x''[t] - ax) -
    0.5*Cd*rho*S*Abs[x'[t] - vx]*(x'[t] - vx);

s = NDSolve[{ode1, ode2, init1, init2, init3, init4}, {x, y}, {t, 0,
    20}];
ParametricPlot[Evaluate[{x[t], y[t]} /. s], {t, 0, 20}]
```

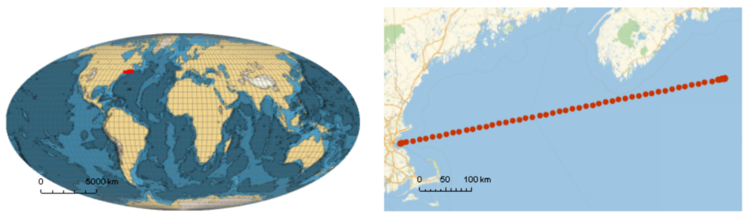
Having done this, I added some functions to the model to represent the position of the drifter on the globe. To do that, I used the Cases function to get the points of the graph, converted these points to latitude, longitude using a function in Python, and then plotted the points using GeoGraphics and GeoListPlot. The code that performs these operations is included below. Also shown below are a couple of maps created by these functions (Figures 12-14).

```
points = Cases[plot, Line[{x__}] -> x, \[Infinity]]
GeoGraphics[{Red, PointSize[.01], Point@GeoPosition@xy},
 GeoRange -> "World", GeoProjection -> "Mollweide",
 GeoGridLines -> Quantity[5, "AngularDegrees"],
 GeoBackground -> "ContourMap"]
GeoListPlot[GeoPosition[xy]]
```



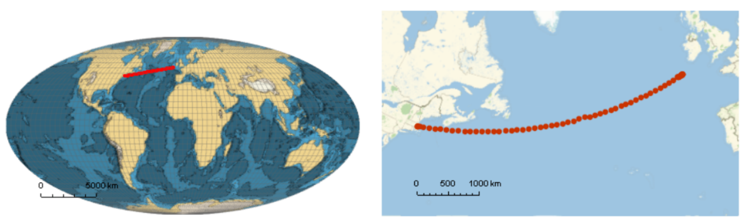
(a) Global view (b) Map view

Figure 12: Position after 2 days



(a) Global view (b) Map view

Figure 13: Position after 20 days



(a) Global view (b) Map view

Figure 14: Position after 150 days

4 Summary

The purpose of this project was to study the motion of drifters. Model drifters were used to measure the coefficient of drag of different shapes, and the wing with multiple small holes proved superior. Subsequently, the coefficient of added mass was calculated, and finally a model was created to predict the drifter's motion in the sea. Given information on the water's velocity and acceleration and the initial position and speed of the drifter, the model creates graphs showing the position of the drifter on a coordinate system and on the globe.

5 Conclusion

This UROP helped offer a better understanding of the motion of drifters, devices widely used but not thoroughly studied. In addition, the model that traces the path can prove useful in the recovery process, in case some of the electronic systems break down. The next steps to enhance the strength of this model would be to incorporate data from the environment, i.e. to mount sensors (e.g. flow meter, accelerometer) on the drifter to calculate the position and acceleration of the water and then update the predicted path accordingly.